

The Global Nature of the Arrow of Time and the Bohm-Reichenbach diagram.

Mario A. Castagnino.
Instituto de Astronomía y Física del Espacio.
Casilla de Correos 67, Sucursal 28,
1428 Buenos Aires, Argentina.

The importance of the global nature of the arrow of time is shown. Classical Reichenbach diagram and quantum Bohm-Reichenbach diagram, for the universe are introduced. They are used to show the increase of entropy in closed systems, the global nature of the quantum measurement, and the relation among the different arrows of time.

I. INTRODUCTION.

This is a conceptual essay about time asymmetry with practically no equations (these equations can be found in other contributions to this volume or in the literature quoted in the bibliography). The main thesis is that, even if the problem of the definition of the local arrow of time would be completely solved (which is not yet the case since there is not an unanimous agreement on the subject), it would not be enough to understand time-asymmetry. In fact, the arrow of time necessarily has a global nature and we sustain that the best structure to explain this arrow of time is a Reichenbach global system. Let us explain these two statements:

II. THE ARROW OF TIME IS GLOBAL.

Let us suppose that the arrow of time would be local. Then, it would be possible to consider two laboratories and to define, in each one, an arrow of time independently. Moreover let us suppose that the two laboratories are perfectly isolated. Then we can ask ourselves: Are the two arrows of time pointing to the same direction? Of course, this question has no answer since, as the two laboratories are isolated, it is impossible to compare one arrow with the other. If we would like to compare the two arrows of time an interaction must be introduced between the two laboratories and we are forced to consider the global system of the two laboratories and the interaction¹. We can repeat the same story if we add a third isolated laboratory and so forth. Then *the* arrow of time will only be well defined if we consider all the possible laboratories and the interactions among each other, namely the whole universe. (See the coincident opinion of Feynman in [1]).

III. REICHENBACH GLOBAL SYSTEM.

The global arrow of time is best represented by a Reichenbach global system ([2], [3] page 127): the system of all branching irreversible processes within the universe, such that any process of the system begins in an unstable state that was produced using energy coming from another process of the global system. E. g.: the famous Gibbs ink drop in the glass of water (initial unstable state) evolves towards a final equilibrium state, the homogeneous mix of ink and water (final stable state) showing that we are dealing with an irreversible process. But the ink drop was not produced by an extremely improbable fluctuation that concentrates the ink in the glass. It was obtained from an ink factory where, to get the necessary energy for the factory coal (initial unstable state) was burnt in an oven until it became ashes (final equilibrium state). *Furthermore the system "ink-water in the glass" only exists as such after the instant when we put the ink drop into the water.* Before this instant a much more complex system exists, that eventually contains the ink factory, the oven, the coal burning, etc. In turn coal was not produced by a fluctuation, quite on the contrary, it was produced using the energy coming from the sun in geological ages. The necessary energy was provided by the light of the sun, where H (initial unstable state) was burnt until it became He and finally Fe

¹From observational evidence we know that all the laboratories in the universe have the same arrow of time. E. g.: if not a radiastronomer would not see the condensation of gas clouds but the opposite, an astronomer would have found stars evolving in a direction opposite to the usual one (Hertzsprung-Russel), or any other sign of the inequality of the local arrows would be detected.

(final equilibrium state). Finally H was produced using the energy coming from the unique initial global state of the whole global system: a cosmological initial instability. This initial unstable state can be explained, after decoupling time, by the effect of the gravitational field that takes the gas and radiation, in equilibrium before that time, into a state of hot condensed clouds of matter surrounded by cold radiation, in an expanding geometry, [4]. If we want to go beyond decoupling time we can consider the nucleosynthesis period [3] or, going closer to the beginning, we can consider Big-Bang quantum cosmological models, which also have an unstable unique initial state [5], [6]. Then through this hierarchical chain, that begins in the cosmological instability and contains all the irreversible processes, where each process begins where the corresponding creation device has finished its task, the irreversible nature of the universe and the origin of any irreversible process in it can be explained. Therefore Gibbs ink drop only exists because there was a primordial cosmological instability and Irreversible Statistical Mechanics can not be explained without Irreversible Cosmology. The global system can be symbolized as in fig. 1, which has a clear time symmetry: the branch arrow of time (BAT), which points in opposite direction to the unique initial cosmological instability and follows the evolution of the hierarchical chain towards equilibrium. Reichenbach global system is clearly a realistic model of the set of irreversible processes within the universe. Let us now see the quantum implication of this idea.

IV. BOHM DIAGRAMS

Let us consider a usual scattering system (with its continuous energy spectrum as those of ref. [7]) and its diagram, with ingoing stable states a_1, a_2, \dots and outgoing stable states b_1, b_2, \dots and a central black box symbolizing any interaction (fig. 2). As it is a reversible process there is no modification of the entropy from the initial to the final states and the eventual evolutions belong to a group. But A. Bohm [7] cuts the box, at a time $t = 0$, in two pieces by a vertical line. The l. h. s. of the this cut figure is a diagram representing the creation of unstable states u_1, u_2, \dots from stable states a_1, a_2, \dots (fig. 3), with a decreasing entropy [8], [9], [10] and a evolution that corresponds to a creation semigroup, for $t < 0$ only [7], [11], [12], [13]. The r. h. s. of fig. 2 is a diagram representing the decaying of unstable states u_1, u_2, \dots ² into stable states b_1, b_2, \dots with a growing entropy that corresponds to a decaying semigroup for $t > 0$ (fig. 4).

The mathematical structures that correspond to the growing and decaying processes can be essentially obtained from the quantum version of Reichenbach idea. Let us first consider the spontaneous decaying states of fig. 4³. It is obvious that these states only exist after the creation time at $t = 0$, because before that time the system was producing growing states. Then the probability to observe a decaying state $|\varphi(t)_-\rangle$ before $t = 0$ in, e. g., any energy out-going eigenstate $|\omega_-\rangle$, namely the out-going Lippmann-Schwinger state, is zero. The classical analogy would be to ask what the probability is to find a particular configuration of the distribution of the ink drop in the glass of water before the ink would be put in the water. This probability is obviously zero since there is no ink in the water, in complete agreement with Reichenbach idea that the irreversible systems *only exist as such after the creation instant*. Then if $t < 0$ we know that $|\langle \omega_- | \varphi(t)_- \rangle|^2 = 0$, therefore also $\int_{-\infty}^{\infty} \langle \omega_- | \varphi(t)_- \rangle d\omega = 0$ or $\int_{-\infty}^{\infty} \langle \omega_- | \varphi(0)_- \rangle e^{-i\omega t} d\omega = 0$. So, from the Paley-Wiener [7] theorem we know that $\langle \omega_- | \varphi(0)_- \rangle \in H_+^2$ the Hardy class from above [7], [14], [15] (see also [16] about the relation of outgoing states and Hardy classes in Lax-Phillips scattering theory). Then if we call Φ_+ the space of states endowed with this last property any decaying state is $|\varphi_-\rangle \in \Phi_+$ and decaying states can be studied considering the Gel'fand triplet $\Phi_+ \subset \mathcal{H}_+ \subset \Phi_+^\times$ where \mathcal{H}_+ is the Hilbert space outgoing states [16] and Φ_+^\times is the space of antilinear functional over Φ_+ . On the other hand, ideal growing states⁴ of fig. 3 cannot exist after $t = 0$. So repeating the above reasoning we can define the space of

²The decaying states are obtained through the interaction of the system, with the continuous spectrum, that plays the role of the "environment" [7]. E. g. the system would be a H atom and the environment would be the electromagnetic radiation that makes all energy levels unstable but the fundamental one [13]. In this sense, even if the whole system is closed (since it contains the atom and the electromagnetic radiation), it evolves as the "open" system of other formalisms. As a branch system it is therefore temporarily isolated, even if we know that a completely isolated system does not exist. But, as explained in [3] (page 125) it is not the interaction with the rest of the universe (e. g. distant galaxies) the one that produces the decaying, but the interaction with the environment just defined.

³In this period the system is quasi-isolated, and theoretically it will be considered completely isolated, as explained in the previous footnote.

⁴These states are just ideal since in the growing period the system is never isolated but it is necessarily receiving energy from a source, i. e. from another branch system within the global Reichenbach system. So these states can only be considered, in the simple model of fig. 2, if we neglect the energy source.

growing states Φ_- as the states $|\varphi_+ \rangle$ such that $\langle \omega_+ | \varphi_+ \rangle \in H_-^2$, the Hardy class from below. Growing states can be studied using the Gel'fand triplet $\Phi_- \subset \mathcal{H}_- \subset \Phi_-^\times$, where \mathcal{H}_- is the Hilbert space incoming [16] states and Φ_-^\times is the space of antilinear functional over Φ_- . Using these mathematical structures the statements about semigroups and entropy can easily be proved [10]. But other mathematical structures can be used instead of the Gel'fand triplet [17], [18], nevertheless the semigroups and the statements about entropy always remain the same. There are other solutions to the problem of the local arrow of time [19], unfortunately the relation and validity of all these solutions is not yet completely understood [9].

V. BOHM-REICHENBACH DIAGRAM.

From the classical Reichenbach diagram of fig. 1 and the Bohm diagrams of the previous section we can obtain a quantum diagram for the universe, first introduced in paper [8], that we will call the Bohm-Reichenbach diagram for the universe, precisely fig. 5. It begins with the cosmological unique primordial unstable decaying state: the r. h. s. cut box in the far left of the figure, followed by all the scattering processes within the universe, all connected among themselves. This global process yields a final thermic equilibrium and therefore a growing of entropy which, defines the thermodynamical arrow of time (TAT), that goes from the unique initial unstable state towards equilibrium. So, from the simple inspection of the figure, we can conclude that $BAT \equiv TAT$.

Entropy also grows in temporally closed and isolated branch systems of the universe ⁵ as the one of the first dotted box (A) of fig. 5, that we reproduce in fig. 6. This would be the simplest closed branch system. It is a scattering process, such that the outgoing states go towards equilibrium, and it also includes the source of energy that it is used to prepare the ingoing state: the r. h. s. cut box in the far left of the figure. The process has an overall growing of entropy due to the decaying process of the initial cut box. Then in any realistic (i. e. containing an irreversible process) closed subsystem of the universe entropy always grows and the Second Law of Thermodynamics is contained, for all closed systems, in Bohm-Reichenbach diagram.

As in the classical case, each irreversible process only exists once the previous creating process has finished its task. Therefore the evolution of these irreversible processes are described by semigroups beginning at the moment of creation of each irreversible process. Therefore the universe evolution is described by a hierarchical chain of irreversible semigroups all based in the same Hardy class, (since all these semigroups are oriented by the BAT, produced by the unique initial unstable states) and not by a reversible group. Even if we have not, by now, the corresponding general mathematical model that would prove of this statement, this scenario was already studied in several papers [20], [21], [22], [23], [24] where a global space Φ_+ is defined for the whole universe. Moreover, it is quite logical that the global Hardy class of the universe would endow with the same properties of analyticity all the local spaces Φ_+ of all the branch systems within the Reichenbach global system.

VI. THE QUANTUM MEASUREMENT PROCESS.

Let us consider a simple example of a measurement process: Stern-Gerlach experiment of fig 7. If we would like to consider the complete preparation-measurement process we must consider not only the scattering process itself, but also the accelerator "A" that prepares the beam, with its source of energy, and the measurement apparatus, namely the detector and counter "B". The accelerator obtains its energy from a source, where a decaying process takes place, and in the detector a creation and a decaying process occurs, e. g.: the particles of the deflected lower beam interact with some atmosphere where some states are excited (creation process) and then they decay ⁶. So the complete process of preparation-measurement corresponds to the dotted box (B) of fig. 5, that we reproduce in fig. 8. Therefore every preparation-measurement process takes place within the Reichenbach global system, since the energy comes from a source that can only be found in this system. Then the preparation procedure turns out to be essentially different from the measurement one: the preparation needs the energy that comes from the hierarchical chain, the measurement is a decaying process, where, even if some part of the energy is used activating the counter,

⁵These systems are similar to those of the previous section, so they have all the features described in the second footnote.

⁶This decaying process takes place in the detector being the environment the atmosphere within the detector. This environment is the one that transform the closed Stern-Gerlach apparatus in an "open" one, in the usual parlance. Anyhow we can consider every thing that is inside box (B) of fig. 5 as a closed system. Again the interaction with the rest of the universe (e. g. distant galaxies) is unimportant.

the rest is degraded. Since the quantum arrow of time (QAT) ([15], [25], [26]), goes from preparation to measurement it necessarily coincides with BAT, so $\text{QAT} \equiv \text{BAT} \equiv \text{TAT}$.

On the other hand, if Reichenbach picture is not used someone would probably say that the difference between preparation state and measurement state is just a technological one i. e.: the outcome of a scattering process is a state very difficult to prepare, but anyhow, it is such that it can be prepared with a highly refined technology. Then QAT would not be an essential asymmetry of nature but just a technological one. This objection disappears if we consider the preparation-measurement process within the Reichenbach system: the difference between preparation and measurement becomes essential: preparation needs energy coming from the primordial instability, independently of the level of technology we use; in the measurement process we do not need energy, which will be degraded in the direction of the final equilibrium state of the universe. Then no confusion is possible between quantum preparation and quantum measurement.

We also see that only in highly idealized scattering processes, with neither preparation nor measurement (fig. 2), the evolution is described by a group. In complete scattering processes, with the preparation and the measurement included, the semigroup structure appears naturally.

VII. THE GEOMETRICAL ARROW OF TIME.

Finally, to further emphasize the global nature of the arrow of time let us consider the case of Classical Relativistic Cosmology. Universe is usually considered to be a time-orientable manifold [27], i. e.: such that all the null semicones can be coordinated, as in fig. 9, in order to be able to define a global arrow of time, that points from the past semicones to the future ones, that we will call the geometrical arrow of time (GAT). If this would not be the case, namely if the manifold could not be oriented, we would have non-causal loops, as $O \rightarrow A \rightarrow O' \rightarrow B \rightarrow O$ of fig. 10 (where the arrows always point towards the future, in some null cone, and nevertheless the cycle begins and ends at O). Then if we like to have a causal universe like ours (where, e. g., measurements always follow preparations) this universe must be a time-orientable manifold and the Reichenbach system, in its relativistic version, must be described in this manifold in such a way that $\text{GAT} \equiv \text{BAT}$. As a manifold can only be either orientable or non-orientable and, as our universe is certainly a time-orientable manifold, it necessarily has a global time-orientation, not a local one, because orientation problems are always global (think in Möbius strip!). So GAT is global and therefore $\text{BAT} \equiv \text{TAT} \equiv \text{QAT}$ must also be global.

The uniqueness of the initial unstable state and the global time orientation of the universe are, therefore, the two bases of the BAT.

We have not yet developed a mathematical structure for this heuristic model, but a first step to introduce this structure is done in papers [28], [29] where the two semigroups are found for the time-like directions of the null cones (one for the future and one for the past) in Relativistic Quantum Mechanics.

VIII. CONCLUSION.

The morale of this essay is therefore that the arrow of time must be considered and defined as a global object in the whole universe, most likely using the Reichenbach global system, for the classical case, or the Bohm-Reichenbach diagram, for the quantum case. Therefore all attempts to define an intrinsic arrow of time based just in local reasonings are necessarily incomplete and probably damned to failure.

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X. FIGURES.

Fig. 1 : The classical Reichenbach diagram.

Fig. 2: A usual scattering diagram.

- Fig. 3: The creation diagram.
 Fig. 4: The decaying diagram.
 Fig. 5: The quantum Reichenbach diagram or Bohm-Reichenbach diagram.
 Fig 6: The diagram of a close subsystem of the universe.
 Fig. 7: Stern-Gerlach experiment.
 Fig. 8: The Bohm-Reichenbach diagram of the Stern-Gerlach experiment.
 Fig. 9: Schematic representation of an orientable manifold.
 Fig. 10: Schematic representation of a non-orientable manifold with an non-causal loop.

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